MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2022/2023

DEM5056 – CALCULUS

(Diploma in Electronics Engineering)

(Diploma in Mechanical Engineering)

3 OCTOBER 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FOUR (4) pages** excluding cover page with **FOUR (4) questions** only.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided. All necessary work **MUST** be shown.
4. Only **non-programmable** calculators are allowed.

QUESTION 1 [25 marks]

(a) A piece-wise function is given as:

$$f(x)=\begin{cases} 0.5x & x<-2 \\ -2x^2+1 & -2\le x<3 \\ 4 & x=3 \\ -x+5 & x>3 \end{cases}$$

(i) Determine the values of $f(-2.5)$, $f(-1)$, $f(3)$, $f(3.9)$ and $f(5)$.

(5 marks)

(ii) Sketch the graph of the function. (4 marks)

(b) A portion of a piecewise function $g(x)$ is shown in **Figure 1** below:

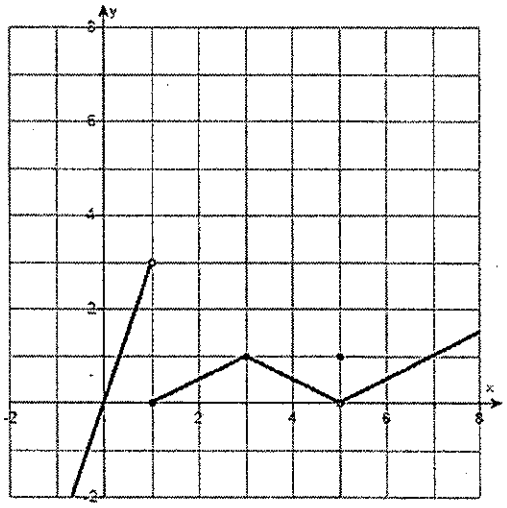

Figure 1

Do the following:

(i) Find the $\lim_{x\to 1} g(x)$ and $\lim_{x\to 3} g(x)$ or explain why it does not exist.

(4 marks)

(ii) Find the $\lim_{x\to 5} g(x)$ or explain why it does not exist. Next, determine $g(5)$.

Check whether the function continuous at $x=5$ and explain why.

(4 marks)

(iii) Find the $\lim_{x\to 7} g(x)$ or explain why it does not exist. Next, determine $g(7)$.

Check whether the function continuous at $x=7$ and explain why.

(4 marks)

Continued...

(iv) If another sub-function of the $g(x)$ when $x > 8$ is defined as $g(x) = \frac{3x}{13x+1}$, find $g(8)$. Next, determine the limit of the sub-function as $x \to -\infty$ and $x \to \infty$. (4 marks)

QUESTION 2 [25 marks]

A polynomial function is given as $y = x^2(4-x)^2$. Do the following:

(a) Show that the local minimum points are $(0, 0)$ and $(4, 0)$, and local maximum point is $(2, 16)$ using 'First Derivative Method'. (16 marks)

(b) Verify the minimum and maximum point(s) obtained in **(a)** using 'Second Derivative Method'. (5 marks)

(c) Determine the interval of increasing and decreasing of the function. (4 marks)

QUESTION 3 [25 marks]

(a) Using suitable integration method, evaluate the following integrals.

(i) $\int_{-2}^{1} (x^3 + 2x - 3)dx$ (3 marks)

(ii) $\int x \sin(3 + x^2)dx$ (5 marks)

(iii) $\int 3x \ln x dx$ (7 marks)

(b) Find the area of the region bounded by the curves $y = x^2 - 4x - 5$ and $y = 5 - x$. (5 marks)

(c) Find the volume of the solid generated when the region bounded by $y = x^2$ and $y = 2 - x^2$, is rotated about x-axis. (5 marks)

Continued...

QUESTION 4 [25 marks]

(a) If $z = \cos y + y\cos x$, where $x = u^2 + v$ and $y = u - v^2$, use the Chain Rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$. (4 marks)

(b) Given $f(x, y) = x^3 + y^3 - 3xy + 5$.

(i) Find the critical point(s) of the function. (4 marks)

(ii) Determine whether the critical point(s) is a maximum, minimum or saddle point. (8 marks)

(c) A lamina occupies a region $D = \{(x, y) \mid -1 \le x \le 1, \quad 0 \le y \le 1\}$ has a density of $\rho(x, y) = x^2$.

(i) Find its mass. (2 marks)

(ii) Find its center of mass. (7 marks)

Continued...

APPENDIX: LIST OF FORMULAE

$$\frac{d}{dx}[\sin x]=\cos x \qquad \frac{d}{dx}[\csc x]=-\csc x\cot x \qquad \frac{d}{dx}[e^x]=e^x$$

$$\frac{d}{dx}[\cos x]=-\sin x \qquad \frac{d}{dx}[\sec x]=\sec x\tan x$$

$$\frac{d}{dx}[\ln x]=\frac{1}{x};\quad x>1$$

$$\frac{d}{dx}[\tan x]=\sec^2 x \qquad \frac{d}{dx}[\cot x]=-\csc^2 x$$

$$\frac{d}{dx}[sin^{-1}x]=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[cos^{-1}x]=-\frac{1}{\sqrt{1-x^2}}\ for-1<x<1$$

$$\frac{d}{dx}[tan^{-1}x]=\frac{1}{1+x^2} \qquad \frac{d}{dx}[cot^{-1}x]=-\frac{1}{1+x^2}\ for-\infty<x<\infty$$

$$\frac{d}{dx}[sec^{-1}x]=\frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}[csc^{-1}x]=-\frac{1}{|x|\sqrt{x^2-1}}\ \ for|x|>1$$

$$\int \tan u\ du=\ln|\sec u|+C \qquad \int \sec u\ du=\ln|\sec u+\tan u|+C$$

$$\int \cot u\ du=\ln|\sin u|+C \qquad \int \csc u\ du=-\ln|\csc u+\cot u|+C$$

$$Area=\int_a^b f(x)-g(x)\,dx \qquad Volume(Washer)=\pi\int_a^b\left[f(x)\right]^2-\left[g(x)\right]^2\,dx$$

$$Volume(Disk)=\pi\int_a^b\left[f(x)\right]^2\,dx$$

$$Volume(Cylindrical\ Shell)=2\pi\int_a^b(shell\ radius)(shell\ height)\,dx$$

Chain Rules:

$$\frac{\partial z}{\partial s}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

Implicit Differentiation:

$$\frac{dy}{dx}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}=-\frac{F_x}{F_y} \qquad \frac{\partial z}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} \qquad \frac{\partial z}{\partial y}=-\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$$

Mass: $m=\iint_D \rho(x,y)dA$ *Center of mass:* $(\bar{x},\bar{y})$ *where,* $\bar{x}=\frac{M_y}{m}$ $\bar{y}=\frac{M_x}{m}$

Moment about x-axis: $M_x=\iint_D y\rho(x,y)dA$

Moment about y-axis: $M_y=\iint_D x\rho(x,y)dA$

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